Tutorial Worksheet 8

1) Which one of the following functions is a bijection? Explain.

$f : \mathbb{R} \to \mathbb{R}$
$f(x) = x^6$

$g : [0, \infty) \to [0, \infty)$
$g(x) = x^6$

$h : \mathbb{Z} \to \mathbb{Z}$
$h(x) = x^6$

$p : \mathbb{Q} \to [0, \infty)$
$p(x) = x^6$

2) Using desmos.com investigate if the following polynomials are injective or surjective. Please note while desmos (or any other graphing tool) can be helpful to make a good observation it is NOT a way to prove any mathematical claims. No proof based on the graph will be accepted in this course. This exercise is for observation only.

a) $p(x) = x^4 - 3x^3 + x^2 - 2x + 5$
b) $q(x) = x^5 + x^2 - 2x + 3$

3) Can we say any polynomial of an odd degree from $\mathbb{R}$ to $\mathbb{R}$ is surjective? Can we say any polynomial of an even degree from $\mathbb{R}$ to $[0, \infty)$ is surjective?

You might need some calculus and the Intermediate Value Theorem, if you are not familiar with it you can skip it.

4) Consider the following functions: $f : \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$, $f(m) = (2m, m-1)$ and $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$
, $g(m, n) = |m.n|$.

(a) Is f injective? Explain.

(b) Is g surjective? Explain.

(c) State the domain and the codomain of $g \circ f$, and write a formula for this composition.

(d) State the domain and the codomain of $f \circ g$, and write a formula for this composition.